SMC 2026 First Round

17th January 2026

1. What is the last digit of the product of all the **odd** numbers between 1 and 2026?

   A. 1 B. 3 C. 5 D. 7 E. 9

2. What is the area of a rectangle with diagonal 25 and width 7?

   A. 32 B. 84 C. 175/2 D. 168 E. 175

3. Let the perimeter and area of a rectangle be P and A respectively. Suppose $1/4 \leq A \leq 4$, what is the minimum possible value of P?

   A. 0 B. 1 C. 2 D. 3 E. 4

4. A kite has diagonals of equal integer lengths. Which of the following is a possible value for its area?

   A. 50 B. 60 C. 80 D. 90 E. 100

5. What is the least positive integer to be multiplied by 2025 to get a perfect $5th$ power?

   A. 3 B. 25 C. 75 D. 125 E. 375

6. Circles of radii 1, 2 and 3 are pairwise externally tangent to each other. What is the area of the triangle formed by the centres of the circles?

   A. 4 B. 5 C. $\sqrt{30}$ D. 6 E. 12

7. The nth term of a sequence is given by $a_n = n + (-1)^n$. What is $a_1 + a_2 + ... + a_{2026}$?

   A. 2053350 B. 2053351 C. 2053352 D. 2055377 E. 2051325

8. How many numbers from 1 to 1000 contain the digit 7.

   A. 142 B. 143 C. 216 D. 271 E. 729

9. Between 00 : 01 and 23 : 59, how many times do the both hands of a clock point in the same direction?

   A. 21 B. 22 C. 23 D. 24 E. 25

10. Let $ABCDE$ be a regular pentagon and let F be an interior point such that DEF is an equilateral triangle. Find the ratio of $\angle AFB$ to $\angle FAB$.

    A. 1 : 1 B. 2 : 1 C. 1 : 2 D. 3 : 2 E. 2 : 3

11. How many integers n such that $\frac{n^6-9}{n^2+1}$ is also an integer.

    A. 3 B. 4 C. 6 D. 7 E. 8

12. Suppose x and y satisfy $x^2 = 4y + 21$ and $y^2 = 4x + 21$. What is the sum of all the possible values of x?

    A. 0 B. 1 C. 4 D. -4 E. -5

13. How many ways can we put the numbers 1, 2, $\cdots$, 10 into five pairs, such that more pairs sum to an even number?

    A. 120 B. 225 C. 345 D. 600 E. 945

14. Let ABC be a triangle and let A' be the reflection of A over BC, let B' be the reflection of B over AC, let C' be the reflection of C over AB. Which of the following **must** be true: (i) $A'B'C'$ is equilateral (ii) $A'B'C'$ is similar to ABC (iii) the circumcicles of $A'BC$, $AB'C$ and ABC' intersect at a common point (iv) the circumcicles of $A'BC$, $AB'C$ and ABC' are of same radius?

    A. (ii) only B. (ii) and (iv) only C. (iii) and (iv) only D. (ii) and (iii) only E. (iv) only

15. A positive integer is *progressive* if its subsequent digits are in strictly increasing order (for example 2578, 34 or 3). How many *progressive* numbers are less than 1000?

    A. 84 B. 90 C. 120 D. 129 E. 132

16. How many $5-digit$ palindromes are divisible by 11?

    A. 45 B. 54 C. 81 D. 82 E. 90

17. Find the number of integers n such that $n^2 + 105$ is a perfect square.

    A. 16 B. 8 C. 4 D. 2 E. 1

18. The distance from a point to the vertices of a rectangle are x, 5, 10 and 11 in no particular order. What are the possible values of x?

    A. 14 B. $1, 7, 14$ C. $2, 7$ D. $2, 14$ E. $2, 14, \sqrt{46}$

19. A number generator generates 1, 2 or 3 with equal probabilities. Pius prompts the generator to generate a random number 5 times. He records these numbers down. What is the probability that the sum of the numbers is even?

    A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{121}{243}$ D. $\frac{122}{243}$ E. $\frac{40}{81}$

20. How many numbers from 1 to 10000 involve at least three different digits?

    A. 7085 B. 8156 C. 8290 D. 9072 E. 9512

21. Given that $xy + ay + ab = 1, yz + bz + bc = 0$ and $xz + cx + ac = 0$. Which of the following is necessarily true?

    A. $a = 0$ B. $x = 0$ C. $y = 0$ D. $y = -b$ E. $z = 0$

22. A unit sphere is dipped in a cone formed from folding a semi-circle of radius 2. How deep does the sphere go?

    A. 1 B. $2 - \sqrt{3}$ C. $2\sqrt{3} - 3$ D. $4 - 2\sqrt{3}$ E. $\sqrt{3} - 1$

23. Solve for $x + y$ in $5x^2 + 2y^2 + 14y + 65 \leq 2xy + 34x$.

    A. $x + y = 1$ B. $x + y \geq 1$ C. $x + y \leq 1$ D. $x + y$ can take any real value E. $x + y$ has no solution.

24. What is the last non-zero digit in 2026!?

    A. 2 B. 4 C. 6 D. 8 E. Cannot be determined

25. In a class, everyone is given a number containing only nonzero digits less than 5. Given any $k \geq 1$ distinct digits less than 5, there are exactly ${}^{10-k}C_2$ people with numbers **containing** all the given k digits. How many people have numbers containing **only** 1?

    A. 0 B. 5 C. 20 D. 36 E. Half of the class