1. A slanted ladder, x unit long, has its top resting on a vertical wall. Suppose the bottom of the ladder (resting on the horizontal ground) is shifted 14 units to the wall, then top of the ladder will move 8 units up the wall. Suppose instead that the bottom of the ladder is shifted 21 units away from the wall, then top of the ladder will move 27 units down the wall. How long is the ladder?

   a) 65 b) 40 c) 81 d) 100 e) 125

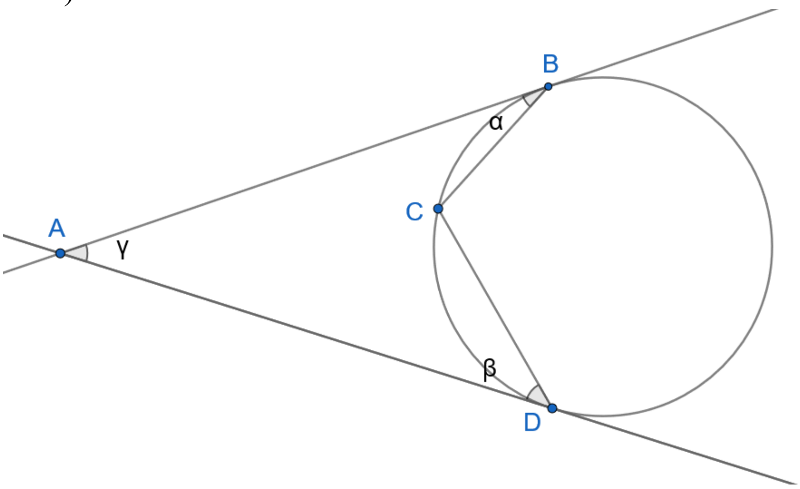
2. $\overline{SMC}_{10}$, $\overline{SCM}_{10}$ and $\overline{CMS}_{10}$ are all perfect squares, where S, M and C are digits (from 0 to 9) of the decimal representation of the numbers. Given that S, M and C are not necessarily distinct, what is the maximum value of $S + M + C$?

   a) 9 b) 12 c) 16 d) 20 e) 27

3. 10 boxes are placed in a row. In how many ways can we put 5 balls, each of a different colour into these boxes if each box can hold at **most** 1 ball and no 2 boxes without balls are adjacent?

   a) 1 b) 4 c) 5 d) 6 e) 252

4. Let $\alpha = A\hat{B}C, \beta = A\hat{D}C, \gamma = B\hat{A}D$ in the diagram below. Given $\alpha + \beta \leq \gamma$, what is the minimum value of γ? (Quadrilateral $ABCD$ is not convex and AB and AD are tangent lines).



   a) 30° b) 45° c) 60° d) 75° e) 90°

5. Simplify $512^{\log_{32} 2023}$.

   a) 2^{2023} b) 16^{2023} c) 2023^4 d) 2023^{16} e) $2023^{9/5}$

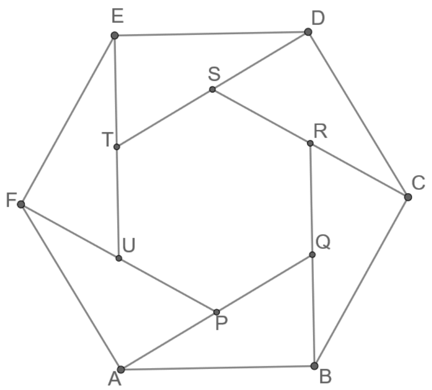
6. What is the probability that in a class of 18 people, there exists a group of 3 people born on the same day of the week?

   a) 0 b) 1/6 c) 5/24 d) 20/21 e) 1

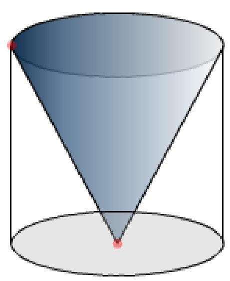
7. In triangle ABC with $|BC| = 72$, let D be the midpoint of $|BC|$ and let E be a point on $|AD|$ such that $|BE| = |BD|$. Suppose E lies in the interior of ABC such that $\angle EBA = \angle DAC$, $\angle EBC = \angle BCA$. Find the length of the altitude from A to BC.

   a) 32 b) 36 c) $24\sqrt{3}$ d) 54 e) $18\sqrt{7}$

8. Find the number of positive integers that are factors of $3^{19}.7^{12}.10^{25}$ but are multiples of $3^{15}.7^{10}.10^{19}$.
   a) 105 b) 735 c) 3240 d) 105360 e) 175760
9. Let $ABCDEF$ be a regular hexagon. Suppose $PQRSTU$ is a hexagon such that $AP = PQ$, $BQ = QR$, $CR = RS$, $DS = ST$, $ET = TU$ and $FU = UP$, as illustrated (P lies on AQ, Q lies on BR, R lies on CS, S lies on DT, T lies on EU, and U lies on FP). Find the ratio of the area of the bigger hexagon to the smaller one.



   a) 2 b) 3 c) 4 d) $2+\sqrt{3}$ e) $2\sqrt{3}$
10. Let a be a zero of the function $f(x) = x^2 - x + 1$. Find the value of $a^{2023} + \left(\frac{1}{a}\right)^{2023}$.
   a) 1 b) 2 c) 2023 d) -1 e) -2
11. A circle of radius 10 with its center at the origin is drawn in the $x - y$ plane. Find the number of lattice points that are on or inside the circle.
   a) 31 b) 37 c) 173 d) 317 e) 371
12. Given positive integers $a_1 \le a_2 \le a_3 \le a_4$, let $F(a_1, a_2, a_3, a_4)$ denote the number of ways of filling a 4 by 4 grid with $a_1 + a_2 + a_3 + a_4$ dots in such a way that each grid contains at most 1 dot and both row k and column k contain exactly a_k dots. Suppose that $F(1,2,2,3) = 24, F(1,2,3,3) = 13$ and $F(1,2,3,4) = 1$. Find $F(1,1,2,3)$.
   a) 1 b) 12 c) 13 d) 24 e) 34
13. What is the remainder when $1^1 + 2^2 + 3^3 + \cdots + 2023^{2023}$ is divided by 7?
   a) 0 b) 1 c) 2 d) 3 e) 4
14. A cylinder is filled to water till it is $\frac{11}{24}$ full. A cone of same height and circular base is submerged in the cylinder as illustrated. What is the height of the water now, as a fraction of the height of the cylinder?



   a) ½ b) 1 c) $^{11}/_{24}$ d) $^{5}/_{6}$ e) $^{5}/_{8}$
15. Find the number of integers greater than **6200** that is formed from the digits 1, 3, 6, 8 and 9, where each digit is used at **most** once.
   a) 54 b) 66 c) 120 d) 186 e) 240

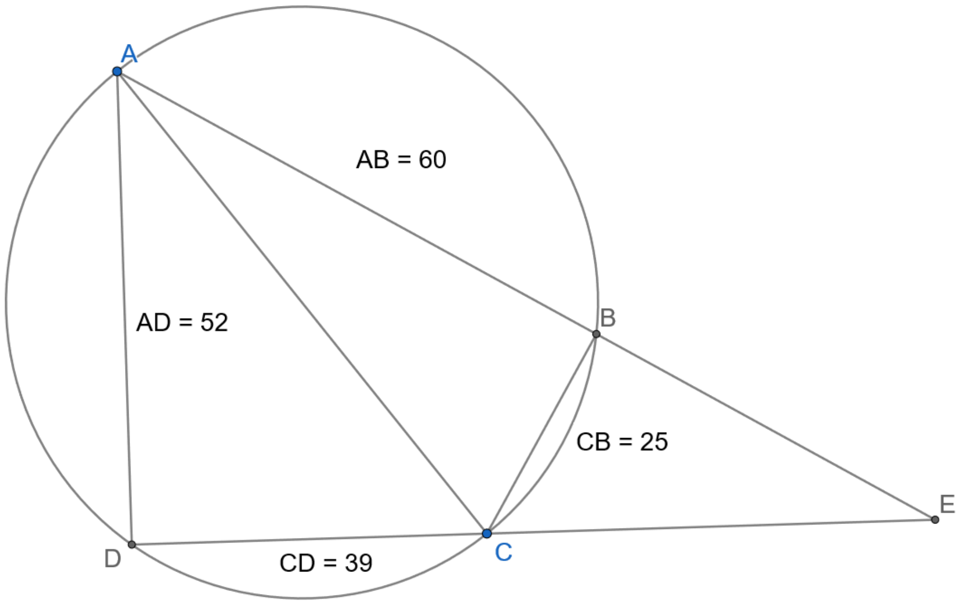
16. Three sequences, a_n, b_n and c_n satisfy $a_n = a_{n+1}a_{n-1}$, $b_n + 1 = b_{n+1}b_{n-1}$ and $c_n = a_{n+1}b_{n-1}$ for all integers n. Find the least value for $T > 0$ such that we **must** have $c_{n+T} = c_n$ for all n.

a) 3 b) 5 c) 6 d) 15 e) 30

17. Which of the following is **not** a factor of $t^{12} - t^7 + t - 1$?

a) $t + 1$ b) $t^2 - t + 1$ c) $t^2 + t + 1$ d) $t^4 - t + 1$ e) $t^6 - t + 1$

18. What is the perimeter of triangle BCE in the diagram below? ($ABCD$ is cyclic)



a) 104 b) 3200/33 c) 176 d) 350/3 e) 1000/11

19. How many ordered pairs of positive integers (m, n) satisfy,

$$\frac{1}{m!} + \frac{1}{n!} = \frac{1}{lcm(m, n)}?$$

a) 1 b) 2 c) 3 d) 4 e) 5

20. What is the sum of the coefficients of terms with even powers of x (terms of the form $x^{2m}y^n$, m and n are integers) in the expansion of $(1 - 2x + y)^{2023}$?

a) 0 b) 2^{4045} c) 2^{4046} d) 2^{2023} e) 2^{2022}

21. Super Eagles coach is tasked with carrying a squad of 23 men for the FIFA's world cup tournament. This squad must consist of at least one goalkeeper, one defender, one midfielder and one attacker. Given that the number of defenders must not be less than the number of midfielders which also must not be less than the number of attackers which also must not be less than the number of goalkeepers, how many possible squad formations are possible? (A possible formation would be 2 goalkeepers, 8 defenders, 7 midfielders and 6 attackers)

a) 94 b) 64 c) 1540 d) 1771 e) 8855

22. An octagon of perimeter 32 units is such that 4 sides have equal lengths of 3 units and the remaining 4 sides have equal lengths of 5 units as well. Order does not matter. Given that this octagon is cyclic, find its circumradius.

a) 4 b) 5 c) $\frac{4+\sqrt{15}}{2}$ d) $\sqrt{\frac{34+15\sqrt{2}}{2}}$ e) $\sqrt{\frac{15+20\sqrt{2}}{2}}$

23. Let a, b, c be positive real numbers such that,

$$a + b + c = 5 \text{ and } \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$$

The range of possible values of c is the real number interval $[e, f]$. Find the value of $f - e$.

a) 1 b) ½ c) 3/2 d) ¾ e) 5/4

24. How many quadruples of positive integers (d_1, d_2, d_3, d_4) are there such that $d_1|5!$, $d_2|5!$, $d_3|5!$, $d_4|5!$ and $gcd(d_1, d_2, d_3, d_4) = 1$? ($a|b$ means a divides b)

a) $2^4 \cdot 3^2 \cdot 5 \cdot 7$ b) $3^2 \cdot 5^4 \cdot 7$ c) $2^3 \cdot 3 \cdot 5$ d) $2^3 \cdot 3^2 \cdot 5 \cdot 7$ e) 2^{16}

25. For all $\binom{2023}{i} = 3^{a_i} \cdot b_i$, $\gcd(b_i, 3) = 1$ with a_i, b_i integers for $i = 0, 1, 2, \dots, 2023$. Find the largest value of i that maximises a_i. ($\binom{n}{k}$ represents the number of ways of selecting k items from n items, gcd (a, b) represents the highest common factor of positive integers a and b)

a) 1943 b) 728 c) 729 d) 1376 e) 1457