<u>Instructions in addition to the ones given in Finalists' Orientation meeting</u>

1. Duration is 4 hours.
2. Use white plane A4 papers for your work and write your name on each paper you submit.
3. Proofs should be provided for all lemmas used and all theorems used should be properly named and stated.

Problem 1: Let $n \in \mathbb{N}$. Prove that

$$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\sqrt{x_i}+\sqrt{y_i}} \geq \frac{\sqrt{n}}{\sqrt{\sum_{i=1}^{n} x_i}+\sqrt{\sum_{i=1}^{n} y_i}}$$

$\forall x_i, y_i \in \mathbb{R}^+, i = 1, 2, \ldots, n.$

Problem 2: Find all triples of prime numbers (p, q, r), such that $q|(r-1)$ and

$$\frac{r(p^{q-1}-1)}{q^{p-1}-1}$$

Is prime.

Problem 3: The numbers 2, 3, 4, …, 100 are written on a board. Chibuike and Ismail play a game of erasing numbers from the board using the following rule: If the number, a, is erased, then only numbers, b, such that $gcd(a, b) = 1$, can be erased, and the person that erased last wins. If Chibuike starts the game, does there exist a winning strategy for him? (Determine with proof.)

Problem 4: Let Ω be a circle with center O and P a point outside it. Tangents from P are drawn to touch the circle at A and B. A point, T, is arbitrarily chosen on major arc AB, and D is the foot of T on AB. K, L, M, N are the midpoints of TA, TB, TD, AB respectively. PT intersects MN at point S. Lines l_a and l_b are the reflections of OA and OB over the angle bisectors of $\angle SAL$ and $\angle SBK$, respectively. Show that l_a, l_b and TD are concurrent.