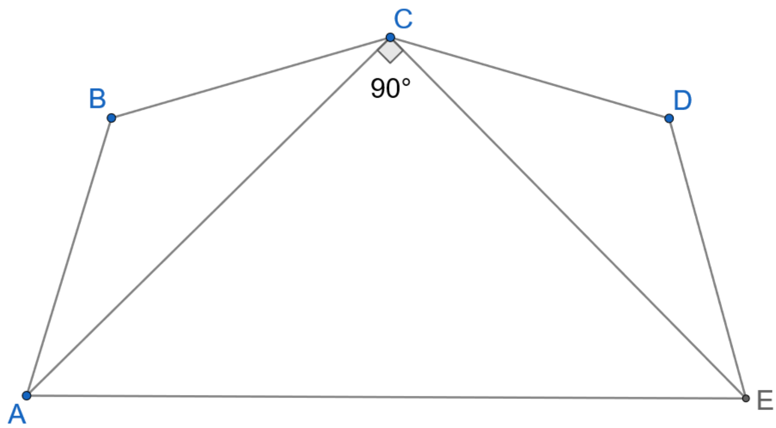
1st Round

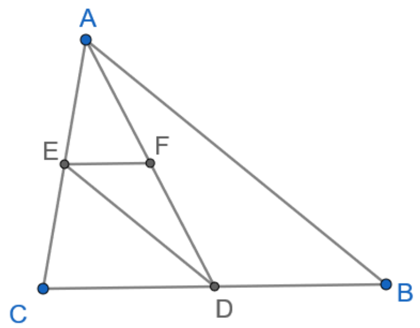
1. From midnight of a certain day, the minute hand of a bewitched clock begins to misbehave in that it moves at same rate but in the opposite direction, while the hour and seconds hands behave normally. When next, after midnight, does the bewitched clock tell the correct time?
   A) 06:00 B) 12:00 C) 15:15 D) 06:30 E) 00:30
2. How many ways can the letters in the word $OLYMPIAD$ be rearranged so that all the vowels form a string of adjacent characters? (Example $LYMPIAOD$)
   A) 4320 B) 40320 C) 43200 D) 4032 E) 432
3. How many 4-digit palindromes are divisible by 7? (A palindrome is a number of the form $abba$)
   A) 7 B) 9 C) 18D) 19 E) 20
4. Given $ABCDE$ is a convex pentagon with $\angle ABC = \angle CDE = 120°$, $\angle ACE = 90°$ and $|AB| = |BC| = |CD| = |DE| = 1$. Find |AE|.



   A) 2 B) $\sqrt{6}$ C) $2\sqrt{2}$ D) $1 + \sqrt{3}$ E) 3
5. Find the last digit of $1^{2^{3^4}} + 2^{3^{4^1}} + 3^{4^{1^2}} + 4^{1^{2^3}}$.
   A) 0 B) 2 C) 4 D) 6 E) 8

6. Given that $8!\,7!\,6!\,5!\,4!\,3!\,2!\,1! = 2^a \cdot b$, where a and b are positive odd integers. What is the value of a?
A) 7 B) 15 C) 23 D) 25 E) 45

7. Given that D is the midpoint of line segment BC, E is the midpoint of line segment AC and F is the midpoint of line segment AD, find the ratio of the area of triangle ABC to triangle DEF.



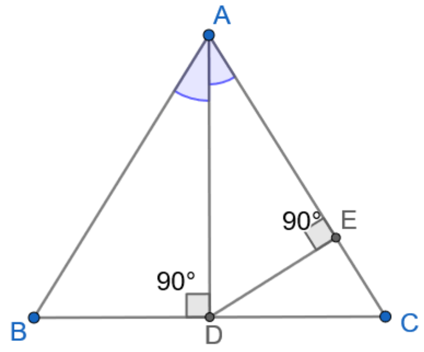
A) 4 B) 6 C) 8 D) $\frac{1}{4}$ E) $\frac{1}{8}$

8. For which of the following pairs $(x,\ y)$ does the inequality
$12x^2 + 6y^2 \geq 17xy$ fail?
A) $(3,\ 5)$ B) $(4,\ 5)$ C) $(4,\ 7)$ D) $(5,\ 7)$ E) $(6,\ 7)$

9. Let p be a prime **greater than** 7, then $(2p - 2)!!\ (mod\ p)$ is? ($x!!$ is double factorial given by $x!! = x(x - 2)(x - 4)... 3\cdot1$, if x is odd and $x!! = x(x - 2)(x - 4)... 4\cdot2$, if x is even)
A) 0 B) 1 C) 2 D) $p - 2$ E) $p - 1$

10. 10 friends arrive at a hotel to spend the night, but the hotel has only 4 rooms free: two 2-man rooms and two 3-man rooms (Rooms are not identical). Given the 10 friends decide to spend the night in the given rooms, in how many ways can they book the rooms?
A) 25200 B) 6300 C) 144 D) 10! E) None of the above

11. In the figure, $|AB| = 25$, $|CE| = 9$, $\angle BAD = \angle DAE$, $AD \perp BC$ at D and $DE \perp AC$ at E. Find $|BC|$.



A) 25 B) 29 C) 30 D) 34 E) Cannot be found

12. Sum of two irrational numbers is 1 less than their product, and 8 less than their sum of squares. Find the larger of the two numbers.

A) $-1-\sqrt{2}$ B) 2 C) 3 D) $-1+\sqrt{2}$ E) $1+\sqrt{2}$

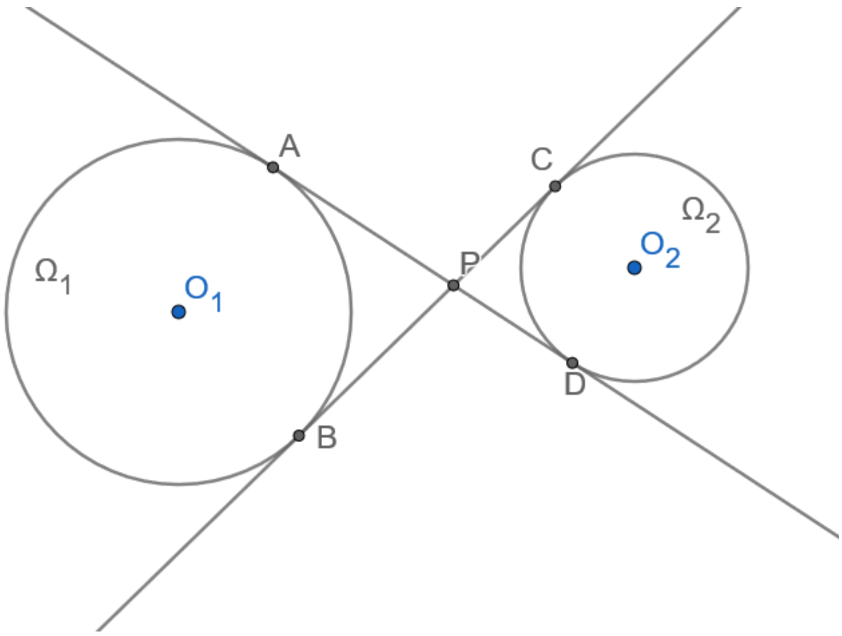
13. In an Olympiad class, half of the students love Geometry (G), half love Number Theory (NT) and half love Combinatorics (C). They all love Algebra. It is also known that 6 love C and NT, 7 love C and G, 8 love G and NT, and 14 love NT but not C nor G. How many students love G but not NT nor C?

A) 12 B) 13 C) 14 D) 15 E) 16

14. An integer n is said to be $k - lengthy$ if it can be written as the sum of k consecutive positive integers. For example, 6 is $3 - lengthy$ as $6 = 1 + 2 + 3$. Suppose that there exists a 3-digit positive integer, m, such that m is $k - lengthy$ and k is as large as possible, find the value of $m + k$.

A) 1079 B) 945 C) 990 D) 1011 E) 1034

15. Let AD and BC be common tangents to circles Ω_1 and Ω_2. Let O_1 and O_2 be the centers of the Ω_1 and Ω_2 respectively. Given that $|O_1O_2| = 15$, $|O_1A| = 6$, $|O_2C| = 3$ and AD intersects BC at P, find $|BP|$.



A) 5 B) 10 C) 7 D) 6 E) 8

16. Consider the sequence of positive integers defined by $a_{n+1} = na_n$ for integers $n \geq 1$. If $a_1 \leq 1000$, find the sum of all the possible number of zeroes a_{2022} can end with.

A) 1512 B) 1007 C) 2018 D) 2525 E) 503

17. A bag contains x blue, y red and z yellow identical balls. Gauss picks 3 balls at random with **replacement**. Given the first two balls are of different colours, what is the probability that the third ball is also of a different colour from the first two?

A) $\frac{1}{3}$ B) $\frac{x!y!z!}{(x+y+z)!}$ C) $\frac{xy+yz+zx}{9xyz}$ D) $\frac{3}{(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$

E) $\frac{6xyz}{(x+y+z)^3}$

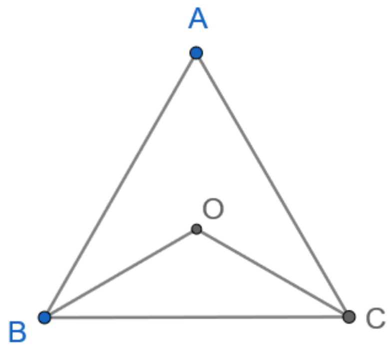
18. Let x, y and n be integers satisfying $nx + (n - 1)y = 1$ and $n > 3$. Then, all of the following are possible except

A) $x = 2n - 1$ B) $y = 2n - 1$ C) $y - x = 2n - 3$ D) $2x + y = 2n - 3$ E) $2y + x = 2n - 3$

19. Given that $f: R \longrightarrow R$ is a non-constant linear function and there exist $a, b \in R$ such that $f(a - b) = a$, $f(a) = b$, $f(b) = a + b - 1$ and $f(a + b) = 1$. Find $f(1)$.

A) 1 B) -3 C) 5 D) 7 E) -9

20. $\angle BAC = \angle CBO = \frac{1}{2}\angle BOC = 50°$, $|AB| = |BC|$. Which of the following is correct?



A) O is the circumcenter of ABC. B) $\angle ABC = 60°$ C) $BO \perp AC$

D) Area of ABC is thrice that of BOC. E) $|AC| = |BO| + |OC|$.

21. John is playing a game with three standard dice. In a single move, he tosses the dices and records the number displayed by each of the dice. John wins the game if the three numbers can be placed side by side to form a 3-digit number divisible by 11. Suppose the probability that John wins is $\frac{p}{q}$ where the fraction $\frac{p}{q}$ is in lowest term. Find $p + q$.
A) 11 B) 5 C) 31D) 29 E) 77

22. Let the polynomial $p(x) = 5x^3 + 3x^2 - 10$ have roots a, b and c. What is the value of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$?
A) $\frac{723}{250}$ B) $-\frac{723}{250}$ C) $\frac{27}{25}$ D) $-\frac{27}{25}$ E) Not possible to be determined

23. 3 positive integers **greater than** 200 are such that the H.C.F of the 1st and 2nd is 143, H.C.F of the 2nd and 3rd is 34, and the H.C.F of the 3rd and 1st is 1. Given the L.C.M of the 3 positive integers is 578578, find the smallest of the 3 positive integers. (H.C.F also known as G.C.D is the Highest Common Factor or Greatest Common Divisor, and L.C.M is the Lowest Common Multiple)
A) 4862 B) 578 C) 286 D) 338 E) 442

24. Let S denote the set of positive multiples of 12, 15 or 20 that are less than 2022. Find the sum of all the elements of S.
A) 409047 B) 403 C) 341727 D) 337 E) 289020

25. Given $P_1P_2P_3 \ldots P_{2022}$ is a convex polygon, what is the value of the following summation?

$$\sum_{i=1}^{i=2022} \angle P_i P_{i+5} P_{i+10}$$

where $P_{2023} = P_1$, $P_{2024} = P_2$, $P_{2025} = P_3$ and so on.

A) $1006 \bullet 360$ B) $2012 \cdot 360$ C) $2013 \cdot 180$ D) $2017 \cdot 360$ E) Not enough information.